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On the structure of pressure fluctuations in turbulent shear flow

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Pressure and pressure–velocity space correlations are calculated, using rapid-distortion theory, for turbulence in a uniform shear flow. It is found that pressure fluctuations remain correlated over significantly greater distances than do velocity fluctuations. When these predictions are used as a model for turbulence in free turbulent shear flows, it is found that the predicted scale of the pressure fluctuations is larger than the flow width. It is proposed that pressure fluctuations remain highly correlated right across free shear flows. Predictions from the theory are then compared with various experimental situations in which reasonable qualitative agreement is to be expected, and this is found.

1. Introduction

Random pressure fluctuations, like velocity fluctuations, are an integral part of turbulent flows, but they have received less attention. This is due mainly to the difficulty of obtaining, except at a solid boundary, reliable measurements in turbulent flows. For flows with solid boundaries there have been numerous theoretical (e.g. Kraichnan 1956; Hodgson 1962) and experimental investigations (e.g. Willmarth & Wooldridge 1962; Bull 1967), particularly on boundary layers. Some early theoretical work was also carried out on pressure fluctuations in isotropic turbulence (Batchelor 1956, p. 117). Space correlations and the ratio $p'/\overline{q^2}$ were calculated, where p' is the r.m.s. pressure fluctuation and $\overline{q^2}$ the sum of the mean squares of the velocity fluctuations. There has, however, been little theoretical work on pressure fluctuations within turbulent shear flows removed from solid boundaries. One exception is the work of Deissler (1962), who considered the development of homogeneous turbulence in a uniform shear flow, but assumed that the turbulence was weak enough for nonlinear interactions to be neglected. However, viscous effects were included. His theory is valid only for turbulence Reynolds numbers much less than one (see below).

Townsend (1970) computed space correlations for turbulent *velocity* fluctuations in a uniform shear flow using rapid-distortion theory, a procedure which should be valid for the large-scale motions that are mainly responsible for the overall shape of correlation functions. His results agreed well with the experimental findings of Rose (1966) for a uniform shear flow. Townsend further hypothesized that, when fluid is entrained into a free turbulent shear flow, "the resultant initial motion is likely to be quasi-isotropic in the sense that it is not highly organized or spatially orientated", and that as "the 'ages' of parcels of turbulent fluid...are usually comparable with local time-scales...the total strain experienced by any parcel is not large compared with one". Hence the turbulence structure should be the result of the finite distortion by the mean shear of the quasi-isotropic turbulence generated by the entrainment process. To test this, he compared velocity correlations predicted from his rapid-distortion theory with those measured in a number of two-dimensional free shear flows. He found very good agreement.

In view of Townsend's success, it appears worthwhile to use the same approach to investigate pressure fluctuations within turbulent shear flows. The main areas of practical interest concerning turbulent pressure fluctuations are jet noise and wall-pressure fluctuations. Although the theory is not applicable to these particular situations, it does provide a reference point for interpreting experimental results from them.

In the next section the theory is presented. Because much of it has been developed previously (Moffatt 1965; Townsend 1970) for velocity fluctuations only those elements which relate to pressure fluctuations are treated in any detail. In § 3, the computed results for space correlations and the ratios $p'/\overline{q^2}$ and p'/τ are presented, τ being the Reynolds shear stress. The theoretical results are discussed in the final section in relation to the somewhat sparse experimental data.

2. Theory for pressure fluctuations

If a turbulent flow with velocity and pressure fluctuations describable by

$$u_i = \sum_{\mathbf{k}} a_i(\mathbf{k},t)\, e^{i\mathbf{k}.\mathbf{x}}, \quad p/\rho = \sum_{\mathbf{k}} \Pi(\mathbf{k},t)\, e^{i\mathbf{k}.\mathbf{x}}$$

is acted upon by a uniform shear such that the mean flow is given by

$$U_1 = (d\alpha/dt)\, x_3, \quad U_2 = U_3 = 0,$$

where $d\alpha/dt$ is the constant strain rate, neglect of nonlinear and viscous terms in the equations of motion and the assumption of incompressibility lead to the following equations for the Fourier coefficients:

$$\begin{aligned}
a_1(\mathbf{k},t) &= a_1(\mathbf{k}_0,0) + \frac{k_0^2}{k_1^2+k_2^2}\left(-\frac{k_2^2}{k_0^2}P + \frac{k_1^2}{k_0^2}Q\right) a_3(\mathbf{k}_0,0),\\
a_2(\mathbf{k},t) &= a_2(\mathbf{k}_0,0) + \frac{k_1 k_2}{k_1^2+k_2^2}(P+Q)\, a_3(\mathbf{k}_0,0),\\
a_3(\mathbf{k},t) &= \frac{k_0^2}{k_0^2 - 2\alpha k_1 k_{30} + \alpha^2 k_1^2}\, a_3(\mathbf{k}_0,0),\\
\Pi(\mathbf{k},t) &= 2ik_1 k_0^2 k^{-4} a_3(\mathbf{k}_0,0),
\end{aligned}$$

where

$$\mathbf{k}_0(0) = (k_1, k_2, k_{30}),$$

$$\mathbf{k} = \mathbf{k}(t) = (k_1, k_2, k_{30} - \alpha k_1), \quad k_{30} = k_3(0),$$

$$P = \frac{k_0^2}{k_1(k_1^2+k_2^2)^{\frac{1}{2}}} \arctan\left(\frac{\alpha k_1 (k_1^2+k_2^2)^{\frac{1}{2}}}{k_0^2 - \alpha k_1 k_{30}}\right),$$

$$Q = \alpha\frac{k_0^2 - 2k_{30}^2 + \alpha k_1 k_{30}}{k_0^2 - 2\alpha k_1 k_{30} + \alpha^2 k_1^2}$$

and α is the total strain.

If one is considering motions with length scale λ and velocity scale v, the nonlinear and viscous terms can be neglected as above if

$$\frac{v}{U_1}, \frac{v}{\lambda\, d\alpha/dt} \ll 1$$

and

$$\frac{\lambda U_1}{\nu}, \frac{\lambda^2 d\alpha/dt}{\nu} \gg 1.$$

That is, the theory is applicable to the larger scales in high Reynolds number, low intensity turbulence.†

Ffowcs Williams (1965) has shown that, for non-zero Mach numbers, compressibility effects influence the largest-scale pressure fluctuations. At sufficiently low Mach numbers, this will be important only for scales so large that they contain negligible energy. The low frequency roll-off (as ω^2) in the surface pressure spectra measured by Hodgson (1962) on the wing of a glider indicates that sufficiently low Mach numbers do occur in practical air flows. This should always be true for water flows.

The equations for the a_i can be written in matrix form:

$$a_i = A_{im} a_m(\mathbf{k}, 0).$$

The three-dimensional wavenumber velocity spectra then become

$$\Phi_{im}(\mathbf{k}, t) = A_{in} A_{mq} \Phi_{nq}(\mathbf{k}_0, 0), \tag{1}$$

after ensemble averaging; in particular

$$\Phi_{33}(\mathbf{k}, t) = (k_0^4/k^4)\, \Phi_{33}(\mathbf{k}_0, 0). \tag{2}$$

The pressure spectrum becomes

$$\Phi_{pp}(\mathbf{k}, t) = 4(k_1^2/k^4)\, (d\alpha/dt)^2\, \Phi_{33}(\mathbf{k}, t) \tag{3}$$

and the pressure–velocity cross-spectra become

$$\Phi_{p3}(\mathbf{k}, t) = 2i(d\alpha/dt)\, (k_1/k^2)\, \Phi_{33}(\mathbf{k}, t) \tag{4}$$

and

$$\Phi_{p1}(\mathbf{k}, t) = 2i(d\alpha/dt)\, (k_1/k^2)\, \Phi_{31}(\mathbf{k}, t). \tag{5}$$

Consider the case when $k_1 \neq 0$ and k is very small in (3). Then from (2)

$$\Phi_{pp}(\mathbf{k}, t) \gg \Phi_{33}(\mathbf{k}, t)$$

for small k, or for large-scale motions. That is, at the large scales there is far more energy in pressure than in velocity fluctuations. This implies that pressure fluctuations remain correlated over larger separations than do velocity fluctuations.‡ It also suggests that the theory should predict pressure correlations at least as well as it predicts velocity correlations.

† The theory of Deissler (1962) assumes that nonlinear terms are negligible but viscous terms are not. Such a theory is true if $v/U_1 \ll 1$, $v[\lambda(d\alpha/dt)]^{-1} \ll 1$ but $\lambda v/\nu \ll 1$. The last condition is a serious restriction and is rarely encountered in turbulent flows of practical significance.

‡ Deissler (1962) also arrived at this conclusion having obtained (3) but his expressions for the $\Phi_{ij}(\mathbf{k}, t)$ are quite different and involve the length scale $(\nu t)^{\frac{1}{2}}$ as an important quantity.

The pressure correlation function $R_{pp}(r_1)$ with streamwise separation is given by

$$\overline{p^2}R_{pp}(r_1) = \int_0^\infty \phi_{pp}(k_1)\cos(k_1 r_1)\,dk_1, \tag{6}$$

where
$$\phi_{pp}(k_1) = 2\iint \Phi_{pp}(\mathbf{k})\,dk_2 dk_3. \tag{7}$$

From (2)
$$\phi_{pp}(k_1) = 8k_1^2(d\alpha/dt)^2\iint (k_0^4/k^8)\,\Phi_{33}(\mathbf{k}_0, 0)\,dk_2 dk_3, \tag{8}$$

but
$$\phi_{pp}(k_1) = \frac{\overline{p^2}}{2\pi}\int_0^\infty R_{pp}(r_1)\cos(k_1 r_1)\,dr_1$$

and so using (8)
$$\phi_{pp}(0) = \frac{\overline{p^2}}{2\pi}\int_0^\infty R_{pp}(r_1)\,dr_1 = 0.$$

This can only occur if $R_{pp}(r_1)$ has a large negative loop. The above relation was also obtained for wall-pressure correlations by Hodgson (1962), who assumed that pressure fluctuations are mainly the result of interaction between the turbulence and the mean shear, which is of course implicit in rapid-distortion theory. It should be noted also that Φ_{pp} is a function of Φ_{33} only, as is Φ_{p3}, while Φ_{p1} is a function of the Reynolds-shear-stress spectrum Φ_{13} only.

Assuming initially isotropic turbulence gives

$$\Phi_{im}(\mathbf{k}_0, 0) = (\delta_{im} - k_i k_m/k^2)\Psi(k_0). \tag{9}$$

$\Psi(k_0)$ is chosen to be

$$\Psi(k_0) = (2/\pi^2)\,\overline{u_{10}^2}\,k_0^2 L^5/(1+k_0^2L^2)^3, \tag{10}$$

implying initial correlation functions of the form

$$R_{im}(\mathbf{r}) = [\delta_{im}(1-\tfrac{1}{2}r/L) + \tfrac{1}{2}r_i r_m/rL]\,e^{-r/L},$$

where $r = |\mathbf{r}|$ and L is termed the 'initial' length scale and is an adjustable constant, and

$$R_{im}(\mathbf{r}) = \overline{u_i(\mathbf{x}+\mathbf{r})\,u_m(\mathbf{x})}/(\overline{u_i^2}\,\overline{u_m^2})^{\frac{1}{2}}.$$

Equation (10) is the form chosen by Townsend (1970) and is a good approximation for large-scale motions in high Reynolds number, grid-generated turbulence.† It is also used here to allow a comparison between the pressure correlations presented below and Townsend's velocity correlations. Equations (1)–(5), (9) and (10) can now be used to calculate space correlation functions and $\overline{p^2}$, using relations similar to (6) and (7). The details of the procedure used are similar to those described in Townsend (1970, appendix B), for velocity correlations, and are not repeated here. From the analysis,

$$\overline{u_i u_m} = \overline{u_{10}^2}\,f_{im}(\alpha)$$

and
$$\overline{p^2} = \rho^2\overline{u_{10}^2}\,L^2(d\alpha/dt)^2 g(\alpha),$$

where f_{i1} and g are functions of α only.

† $\phi_{pp}(k_1)$ was calculated using (9) and $\Psi(k_0) = k_0^2L^5(32\pi^3)^{-\frac{1}{2}}\exp(-\tfrac{1}{2}k_0^2L^2)$. There was no difference at the low wavenumber end of the spectrum.

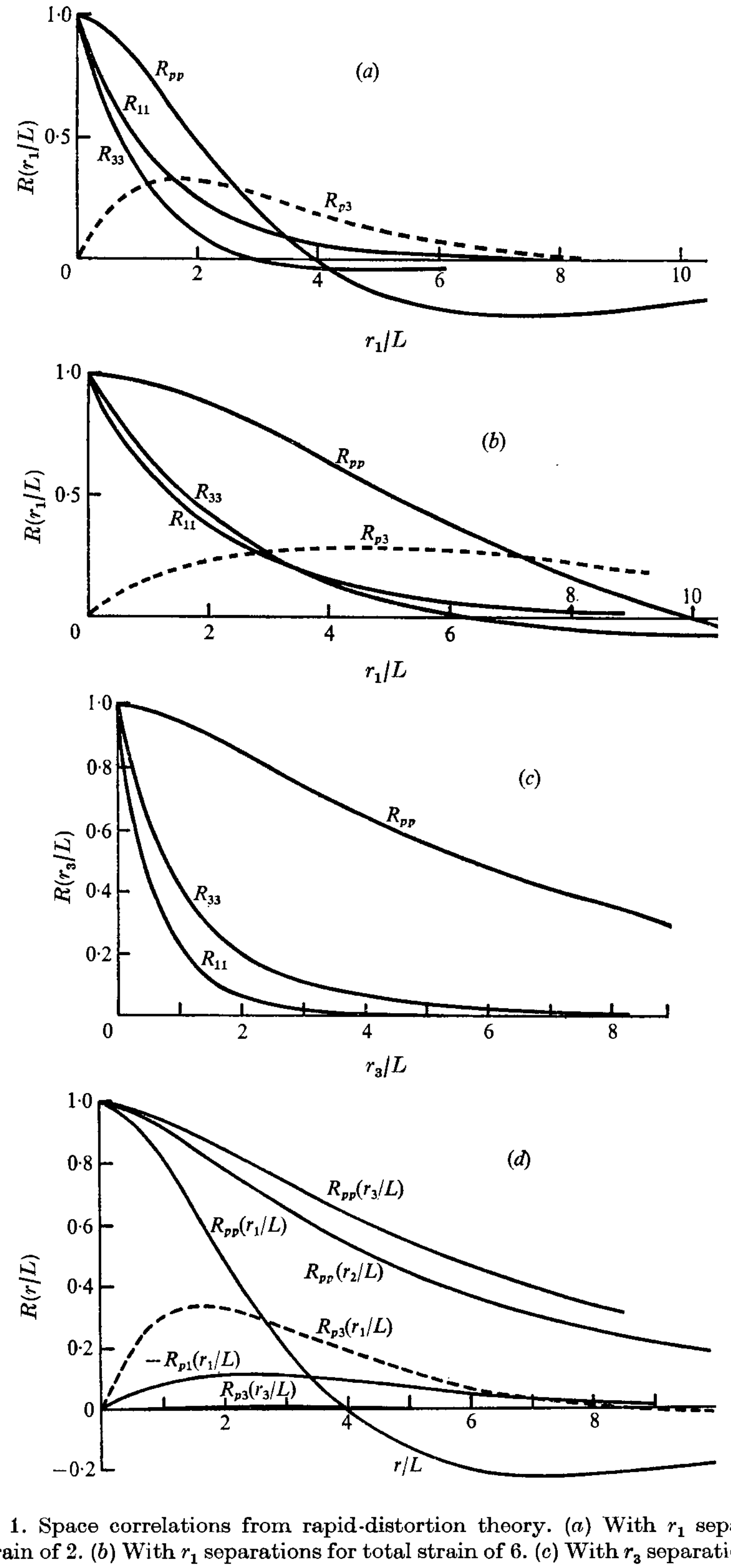

FIGURE 1. Space correlations from rapid-distortion theory. (*a*) With r_1 separations for total strain of 2. (*b*) With r_1 separations for total strain of 6. (*c*) With r_3 separations for total strain of 2. (*d*) With separations along various axes for a total strain of 2.

3. Theoretical results

In figure 1 (*a*), $R_{pp}(r_1/L)$ is compared with the velocity correlations $R_{11}(r_1/L)$ and $R_{33}(r_1/L)$, for a total strain of 2. As was inferred in the previous section from (3), the pressure fluctuations are correlated over significantly greater separations than velocity fluctuations and $R_{pp}(r_1/L)$ has a large negative loop. The greater spatial scale of pressure fluctuations is also illustrated in figure 1 (*b*) for r_1 separations at a total strain of 6 and in figure 1 (*c*) for r_3 separations at a total strain of 2. Figures 1 (*a*) and (*b*) also show the correlation $R_{p3}(r_1/L)$ for comparison with the other curves.

In figure 1 (*d*), pressure correlations for separations in the three orthogonal directions and some pressure–velocity correlations are grouped for comparison with each other. It can be seen that $R_{pp}(r_2/L)$ and $R_{pp}(r_3/L)$ remain positive and that $R_{p3}(r_1/L)$ is about three times as large as $R_{p1}(r_1/L)$. Most of the curves presented are for a total strain of 2, to allow comparison with Townsend's results.

The space correlations $R_{pp}(\mathbf{r}/L)$, $R_{p3}(\mathbf{r}/L)$ and $R_{p1}(\mathbf{r}/L)$ for separations in the r_1, r_3 plane are presented in figure 2 in the form of isocorrelation curves. Their asymmetry about the axes is marked. Those for separations in the r_1, r_2 plane are presented in the same form in figure 3. Because of the symmetry properties of the mean flow (Craya 1958), $R_{pp}(\mathbf{r}) = R_{pp}(-\mathbf{r})$, $R_{pi}(\mathbf{r}) = -R_{pi}(-\mathbf{r})$ and the correlations R_{pp}, R_{p1} and R_{p3} are invariant under reflexion about the r_1, r_3 plane. Hence only half the r_1, r_3 plane and a quarter of the r_1, r_3 plane need be presented. Also, from the symmetry properties of the mean flow, $R_{p2}(\mathbf{r}) = 0$ if $r_2 = 0$; that is, in the r_1, r_3 plane. $R_{p2}(\mathbf{r})$ has not been calculated for separations in the r_1, r_2 plane.

Figure 4 presents the non-dimensional groups $(p'/\tau)\beta$ and $(p'/\rho\overline{q^2})\beta$ *vs.* the total strain, where $\beta = (\overline{u_1^2})^{\frac{1}{2}}/(L d\alpha/dt)$. In §2, it was seen that $v/(\lambda\, d\alpha/dt) \ll 1$ for the theory to apply, and β is really the 'initial' value of this number. For a total strain of 2 it can be seen that $(p'/\rho\overline{q^2}) = 0{\cdot}06/\beta$. For $\beta^{-1} \simeq 20$, $p'/\rho\overline{q^2} \simeq 1{\cdot}2$. This may be compared with $p'/\rho\overline{q^2} = 0{\cdot}19$, calculated for isotropic turbulence (Batchelor 1956, p. 182). In this paper p', $\overline{q^2}$ and τ have been calculated using equation (10) for $\Psi(k_0)$. Townsend (1970) calculated $\overline{q^2}$ and τ using

$$\Psi(k_0) = k_0^2 L^5 (32\pi^3)^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}k_0^2 L^2\right).$$

Townsend's and the author's calculations of $\tau/\rho\overline{q^2}$ are compared in the lower part of figure 4. Experimental points from uniform shear flows for $\tau/\rho\overline{q^2}$ are also shown. The information on $\tau/\rho\overline{q^2}$ is presented to demonstrate the order of variation in the predictions with changes in initial conditions and the degree of agreement with experiments, for this particular ratio. Similar variations are to be expected for the ratios p'/τ and $p'/\rho\overline{q^2}$.

The calculations also show that at the instant $t = 0$ there is a pressure fluctuation field and $R_{pp}(\mathbf{r})$ and $R_{pi}(\mathbf{r})$ are non-zero. This implies that pressure fluctuations are set up at the instant the strain field is applied.

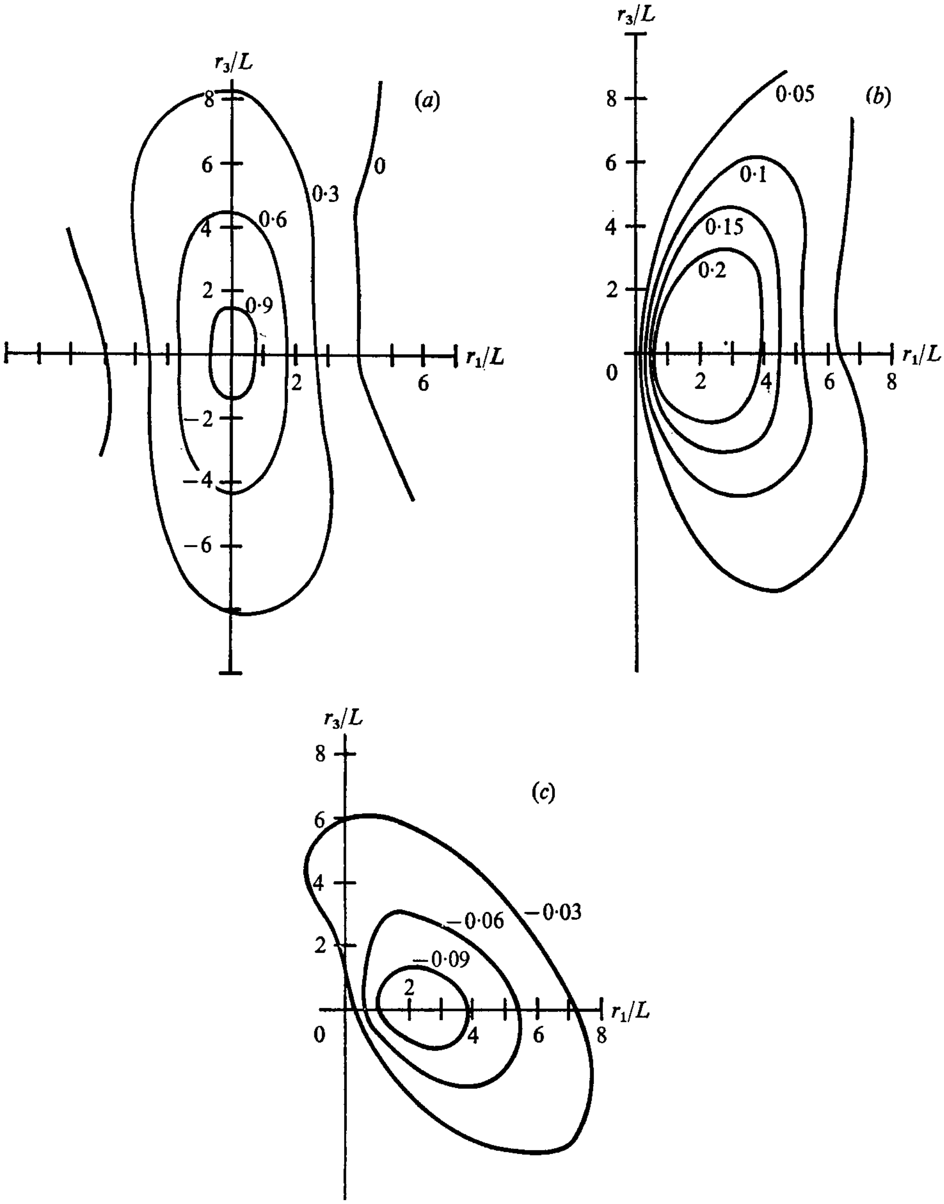

FIGURE 2. Isocorrelation curves for separations in the r_1, r_3 plane for a total strain of 2. (*a*) $R_{pp}(\mathbf{r}/L)$, (*b*) $R_{p3}(\mathbf{r}/L)$, (*c*) $R_{p1}(\mathbf{r}/L)$.

4. Discussion

Limitation of flow width on lateral scales

Townsend (1970) compared computed velocity correlations in detail with the measurements by Rose (1966) in a uniform shear flow and by Grant (1958) in a two-dimensional wake. He found very good agreement. In Rose's experiments the distance between the measuring station and the duct side walls was $6L$, and

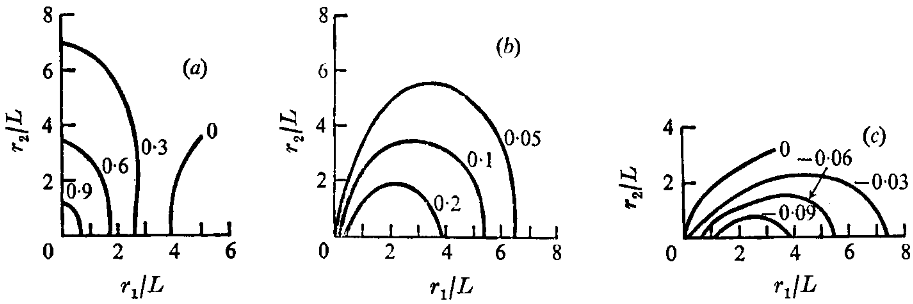

FIGURE 3. Isocorrelation curves for separations in the r_1, r_2 plane for a total strain of 2. (*a*) $R_{pp}(\mathbf{r}/L)$, (*b*) $R_{p3}(\mathbf{r}/L)$, (*c*) $R_{p1}(\mathbf{r}/L)$.

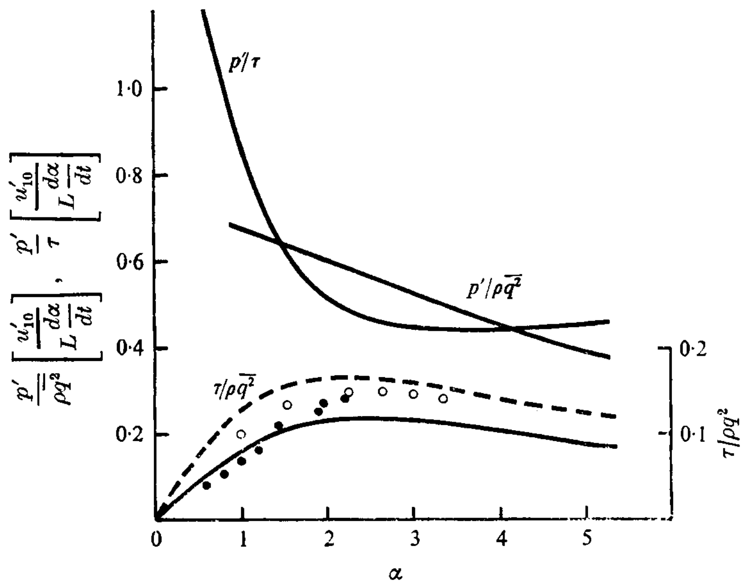

FIGURE 4. Variation of $\beta p'/\tau$, $\beta p'/\rho\overline{q^2}$ and $\tau/\rho\overline{q^2}$ with total strain α. $\beta = (u_1^2(0))^{\frac{1}{2}}/(L d\alpha/dt)$. ———, theory using initial conditions of equation (10); – – – –, theory using Townsend's (1970) initial conditions; ○, experimental results of Champagne *et al.* (1970); ●, experimental results of Rose (1966).

in Grant's experiments the edge of the wake was approximately $3{\cdot}5L$ from the measuring position. Figures 1–3 show that for lateral separations of this order the values of the pressure and pressure–velocity correlations are very high. For the experiments in a uniform shear flow, this suggests that the duct used by Rose was not wide enough. It may be that the theory as a whole is inapplicable to free shear flows and that the remarkably good agreement found for velocity correlations is coincidental. This, however, appears unlikely. It is more probable that pressure fluctuations do remain highly correlated right across turbulent shear flows, as sketched in figure 5. Thus, although the pressure and pressure–velocity correlations predicted for r_3 separations cannot be correct, the departures from these predictions, within the turbulent region, may not be important for velocity

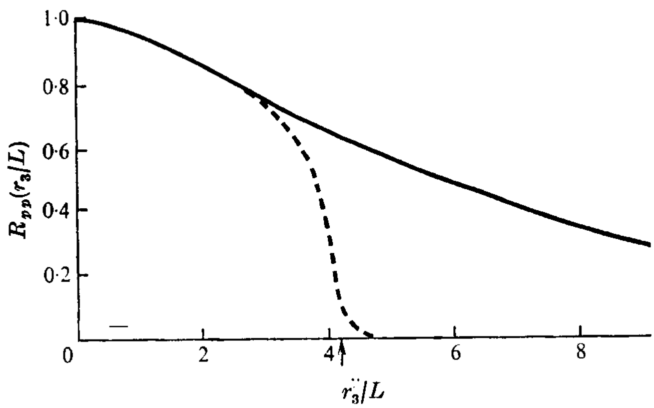

FIGURE 5. Pressure correlation across a shear flow. ——, theoretical curve; – – – –, conjectural experimental curve. Arrow indicates free edge of shear flow.

correlations or pressure correlations with separations in the r_1, r_2 plane. Similarly, the measurements from experiments in a uniform shear flow, not only those by Rose (1966) but also those by Champagne, Harris & Corrsin (1970) and Mulhearn & Luxton (1975), may have been little affected by limited duct size.

Experimental evidence

The author knows of no experimental results from two-dimensional shear flows, with high Reynolds number and low intensity, with which the theory can be compared and the above conjectures checked. There is some indirect corroboration from results in a two-dimensional mixing layer, from boundary-layer wall-pressure fluctuation data and from atmospheric data. In all these cases nonlinear effects are not negligible but one would expect some qualitative agreement, especially with regard to the relative scales of pressure and velocity fluctuation fields. Measurements of pressure fluctuations within turbulent flows must be treated with some caution, owing to the many possible sources of error (see Siddon 1969). For instance, if the static pressure sensing orifices are not located on the probe body at a position where the mean static pressure is the same to a high degree of accuracy as the true value, then streamwise velocity fluctuations will give rise to substantial errors. It appears likely that the measurements made by Kobashi (1957) and Strasberg (1963) of turbulence in the near wakes of circular cylinders suffered from this problem.

Jones (1967) attempted to measure pressure fluctuations generated in the mixing layer of a two-dimensional jet. He used a probe microphone arrangement, the probe tip having a rounded end and the static pressure holes being 10 probe diameters (or 12 mm) from the nose. The probe thus correctly sensed only the larger-scale motions. Also, the measured signal must have been contaminated by transverse velocity fluctuations. The measurements he obtained were auto-correlations not space correlations. If one uses Taylor's hypothesis (a crude approximation for this flow), the theoretical space correlations and experimental autocorrelations may be compared. Figure 6 shows such a comparison for

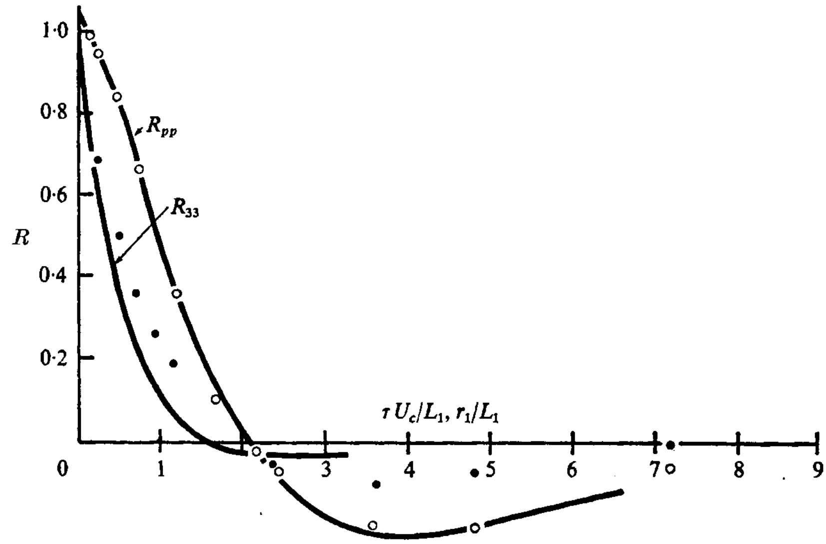

FIGURE 6. Comparison between theory and experiment for a two-dimensional mixing layer. ——, theoretical curves. Experimental points of Jones (1967): ○, $R_{pp}(\tau U_c/L_1)$; ●, $R_{33}(\tau U_c/L_1)$. (L = 9·4 mm, L_1 = 17·3 mm.)

$R_{pp}(r_1)$ and $R_{33}(r_1)$, with $L = 9{\cdot}4$ mm. L_1 on this figure is the area under the $R_{11}(r_1)$ curve to the first zero crossing. These measurements were obtained at $x_1/D = 2{\cdot}5$, $x_3/D = 0{\cdot}05$, with the origin at the tip of the nozzle. D is the nozzle width. The convection velocity U_c was taken from Bradshaw, Ferriss & Johnson (1964) as 0·6 times the free-stream velocity. The theoretical curves are for a total strain of 2. The total strain of the flow was estimated to have approximately this value from TdU_1/dx_3, where T is the time delay after which the moving-axis autocorrelation for the u_1 fluctuations has decreased to e^{-1}. ($R_{33}(r_1)$ and the moving-axis autocorrelation function were supplied privately to the writer by Dr I. S. F. Jones.) It can be seen that agreement, so far as the relative scale of velocity and pressure fluctuations is concerned, is good.

Jones corrected his r.m.s. pressure measurements for the effects of transverse velocity fluctuations. If this corrected value is used,

$$p'/\rho\overline{q^2} \simeq 0{\cdot}4,$$

again at $x_1/D = 2{\cdot}5$, $x_3/D = 0{\cdot}5$. The ratio $(\overline{u_1^2}(0))^{\frac{1}{2}}/(Ld\alpha/dt)$, which is needed to compare this value with the theory, is hard to obtain because of the difficulty of estimating $(\overline{u_1^2}(0))^{\frac{1}{2}}$. If the value of u_1' at the edge of the mixing layer, where the mean velocity just equals that in the potential core, is used, then $(u_1^2(0))^{\frac{1}{2}}/(Ld\alpha/dt) \doteq 0{\cdot}14$. Therefore the predicted value for a total strain of 2 is

$$p'/\rho\overline{q^2} = 0{\cdot}435.$$

That the agreement is so remarkably good is probably coincidental but the theoretical value is clearly of the right order.

Many measurements have been made of turbulent boundary-layer wall-pressure fluctuations. Larger-scale wall-pressure fluctuations are influenced by

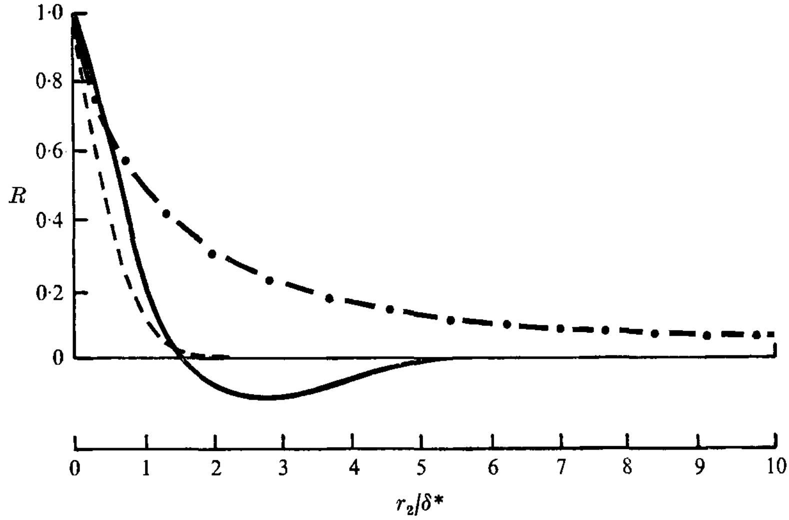

FIGURE 7. Comparison of velocity correlations in the outer part of a turbulent boundary layer with the pressure correlation at the wall, all having transverse separations. ———, R_{11}, Grant (1958); – – – –, R_{33}, Grant (1958); — ● —, R_{pp}, Bull (1967).

both the outer and the inner parts of the boundary layer, the scale of the velocity fluctuations being larger in the outer part. In figure 7, velocity correlations (from Grant 1958) measured in the outer part of a turbulent boundary layer are compared with wall-pressure correlations (from Bull 1967), both correlations having transverse (r_2) separations. The pressure fluctuations retain significant correlation to a separation twice as large as that for the velocities. If one recalls that the wall pressures will be markedly affected by the inner part of the boundary layer, the larger scale predicted for the pressure fluctuations receives increased support.

Correlations between turbulent boundary-layer wall-pressure and velocity fluctuations have been measured by Willmarth & Wooldridge (1963; discussed in Favre 1965) and Burton (1971). These references show $R_{p1}(\mathbf{r})$ and $R_{p3}(\mathbf{r})$ to be of the same order and of opposite sign. The shapes of the experimental curves agree with those predicted, but the relative magnitude of R_{p3} and R_{p1} does not. This discrepancy is most likely due to poor prediction of the denominators $(\overline{p^2}\,\overline{u_i^2})^{\frac{1}{2}}$ of the correlation functions. Much of the energy of the velocity fluctuations resides at wavenumbers where rapid-distortion theory does not apply. A consequence is that the ratio u_3'/u_1' is drastically underestimated because the non-linear terms neglected by the theory decrease anisotropy (Mulhearn 1974). If u_3' and u_1' are more nearly equal in magnitude, so will R_{p1} and R_{p3} be.

Both in turbulent boundary layers (e.g. Kline *et al.* 1967) and in mixing layers (e.g. Brown & Roshko 1974) there have been extensive experiments aimed at investigating the large scales of the turbulence in terms of coherent structures, rather than from the statistical viewpoint of this paper. The two approaches should be compatible, even if it is not clear how they relate at this stage.

Pressure fluctuations within the lower layers of the atmosphere and at the ground were measured by Elliott (1972). He found that pressure spectra were independent of height and that there was no difference in phase between pressures measured at different heights. This was over height intervals of up to 5·5 m. This behaviour contrasts with that of velocity fluctuations, whose spectra scale with height and whose cross-spectra, for vertical separations, display a phase shift. Again these measurements support the predicted difference in scale between pressure and velocity fluctuations.

5. Conclusions

Pressure and pressure–velocity correlations were calculated from rapid-distortion theory for a uniform shear flow. Because of the success of Townsend (1970) in predicting velocity correlations it appeared plausible to use these results to model the pressure field in two-dimensional free shear flows. The lateral scale predicted for the pressure fluctuations was larger than the flow width. In view of the good agreement previously found for velocity fluctuations it was proposed that pressure fluctuations are highly correlated right across a turbulent shear flow. To support the predictions of the theory and, by implication, the last proposition, data from experiments with which one would expect at least qualitative agreement were examined, and the degree of agreement found was encouraging. A proper test of the theory would require measurement of space correlations within turbulent flows. It might be worthwhile to attempt this using the probe described by Siddon (1969) and discussed in Willmarth (1971).

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